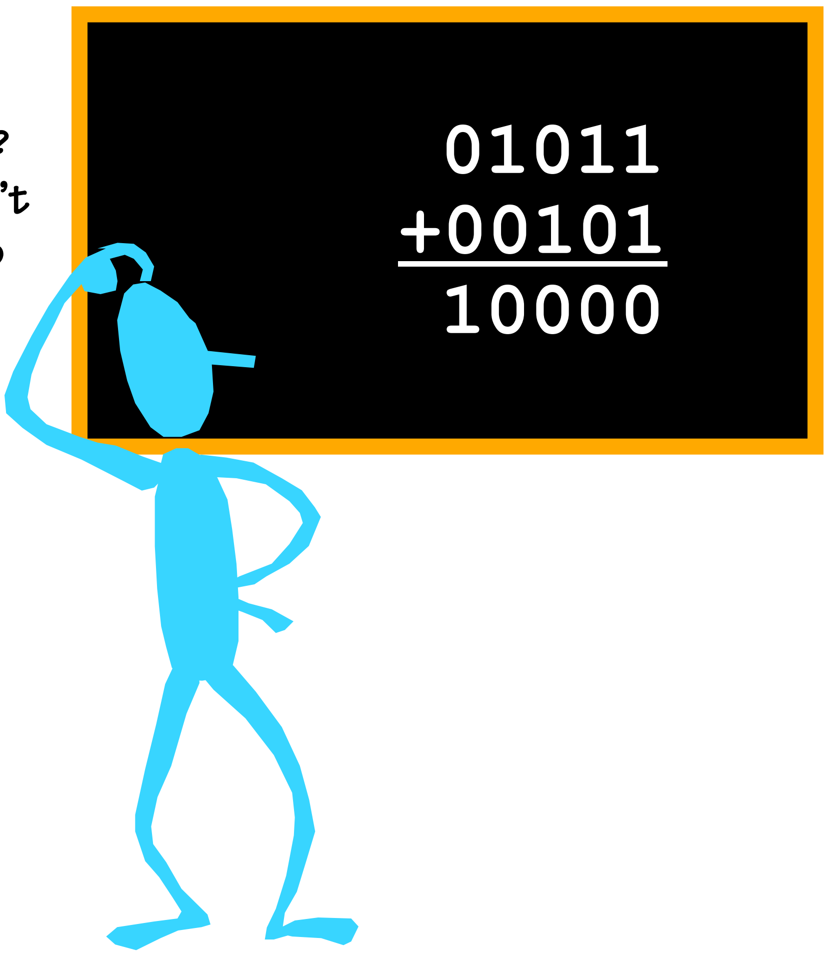
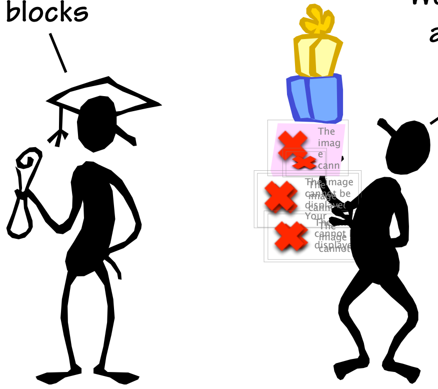
Arithmetic Circuits

Didn't I learn how to do addition in the second grade? UNC courses aren't what they used to be...



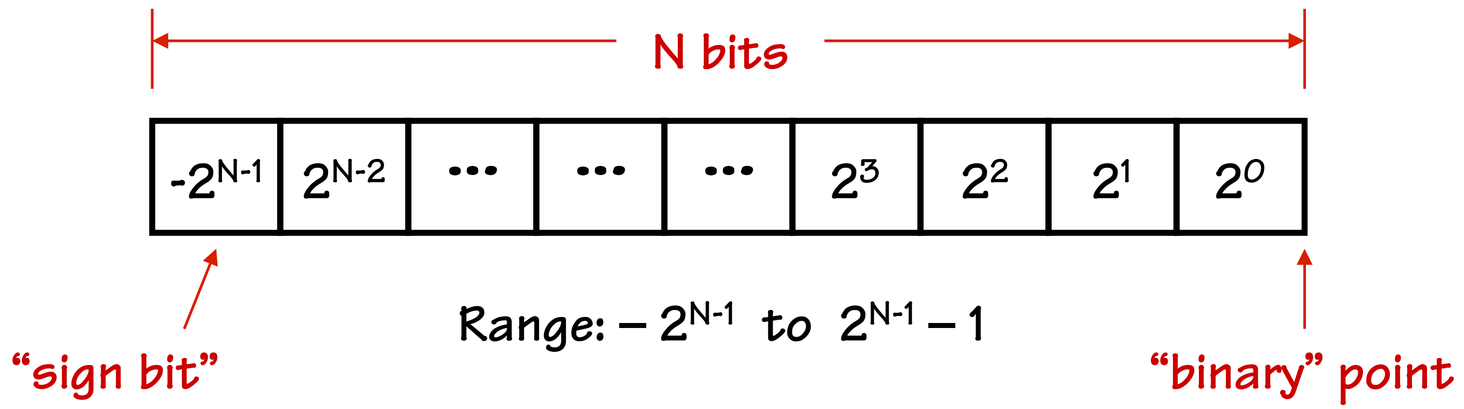
Finally; time to build some serious functional blocks

We'll need a lot of boxes



Reading: Study Chapter 3.

Review: 2's Complement



8-bit 2's complement example:

$11010110 = -2^7 + 2^6 + 2^4 + 2^2 + 2^1 = -128 + 64 + 16 + 4 + 2 = -42$

If we use a two's-complement representation for signed integers, the same binary addition procedure will work for adding both signed and unsigned numbers.

By moving the implicit "binary" point, we can represent fractions too:

$1101.0110 = -2^3 + 2^2 + 2^0 + 2^{-2} + 2^{-3} = -8 + 4 + 1 + 0.25 + 0.125 = -2.625$

Binary Addition

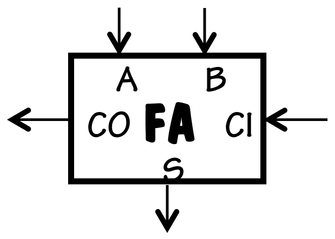
Here's an example of binary addition as one might do it by "hand":

```
        1 1  0 1
A:      1101
B:+     0101
       10010
```

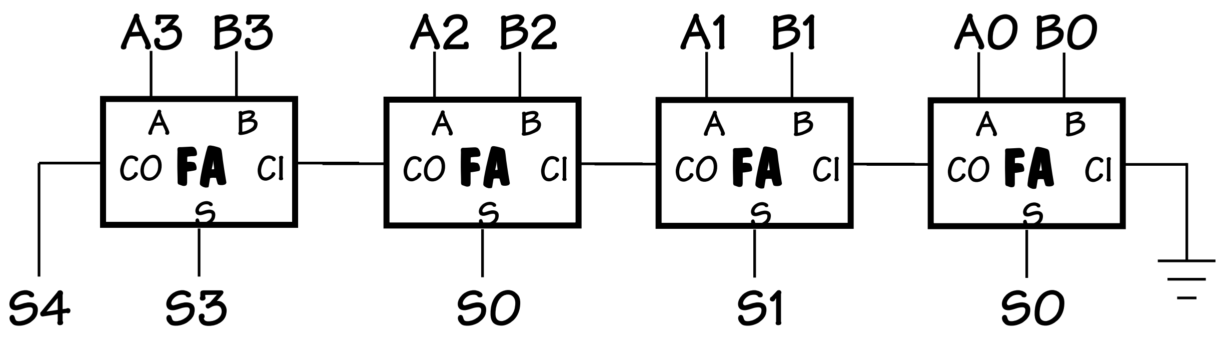
Carries from previous column

Adding two N-bit numbers produces an (N+1)-bit result

Let's start by building a block that adds one column:



Then we can cascade them to add two numbers of any size…



Designing a Full Adder: From Last Time

1) Start with a truth table:

C_i	A	B	C_o	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

2) Write down eqns for the "1" outputs

$$C_o = \overline{C_i}AB + C_i\overline{A}B + C_iA\overline{B} + C_iAB$$
$$S = \overline{C_i}\,\overline{A}B + \overline{C_i}A\overline{B} + C_i\overline{A}\,\overline{B} + C_iAB$$

3) Simplifing a bit

$$C_o = C_i(A + B) + AB$$
$$S = C_i \oplus A \oplus B$$

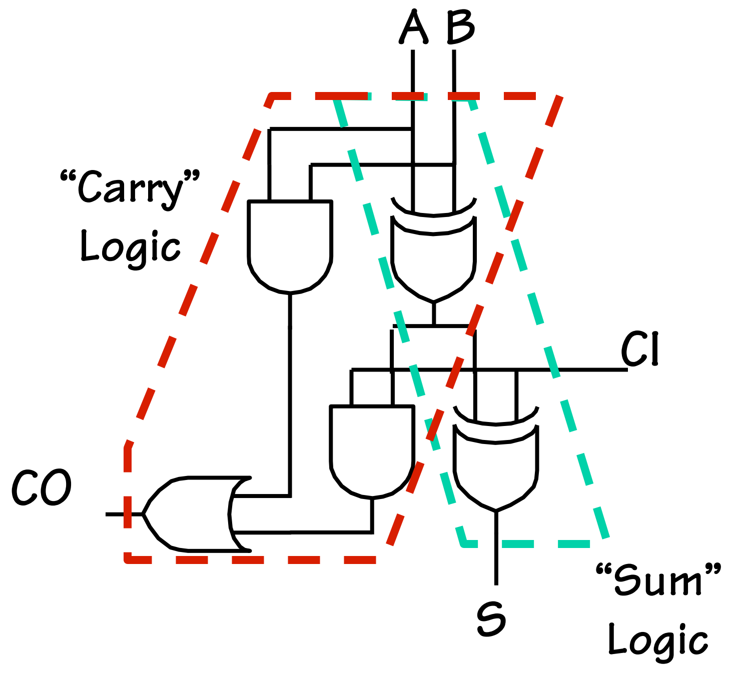
$$C_o = C_i(A \oplus B) + AB$$
$$S = C_i \oplus (A \oplus B)$$

For Those Who Prefer Logic Diagrams …

$C_o = C_i(A \oplus B) + AB$

$S = C_i \oplus (A \oplus B)$

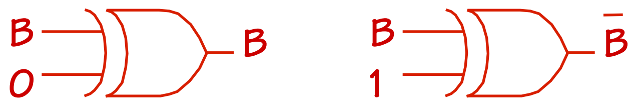
- A little tricky, but only 5 gates/bit



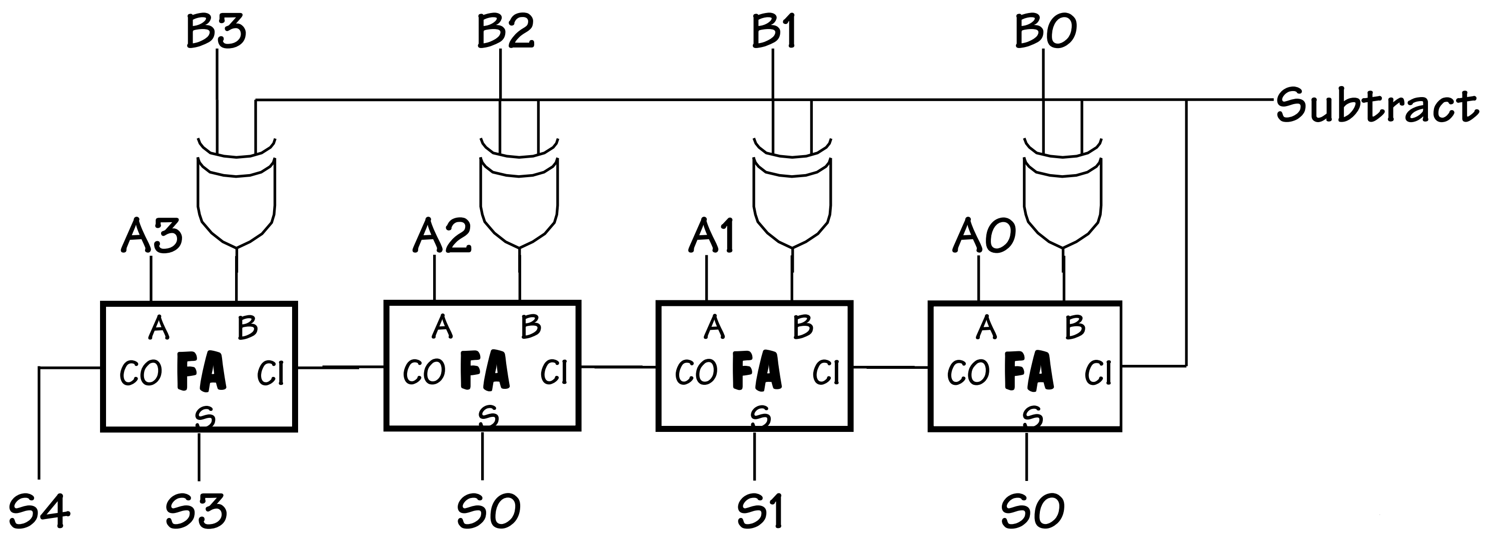
Subtraction: A-B = A + (-B)

Using 2's complement representation: –B = ~B + 1

~ = bit-wise complement



So let's build an arithmetic unit that does both addition and subtraction. Operation selected by control input:



Condition Codes

Besides the sum, one often wants four other bits of information from an arithmetic unit:

Z (zero): result is = 0 *big NOR gate*

N (negative): result is < 0 S_{N-1}

C (carry): indicates that add in the most significant position produced a carry, e.g., "1 + (-1)" *from last FA*

V (overflow): indicates that the answer has too many bits to be represented correctly by the result width, e.g., "$(2^{i-1} - 1) + (2^{i-1} - 1)$"

$$V = A_{i-1} B_{i-1} \overline{N} + \overline{A}_{i-1} \overline{B}_{i-1} N$$

-or-

$$V = CO_{i-1} \oplus CI_{i-1}$$

To compare A and B, perform A–B and use condition codes:

Signed comparison:

LT	$N \oplus V$
LE	$Z + (N \oplus V)$
EQ	Z
NE	$\sim Z$
GE	$\sim (N \oplus V)$
GT	$\sim (Z + (N \oplus V))$

Unsigned comparison:

LTU	C
LEU	$C+Z$
GEU	$\sim C$
GTU	$\sim (C+Z)$

Shifting Logic

Shifting is a common operation that is applied to groups of bits. Shifting can be used for alignment, as well as for arithmetic operations.

X << 1 is approx the same as 2*X

X >> 1 can be the same as X/2

For example:

$$X = 20_{10} = 00010100_2$$

Left Shift:

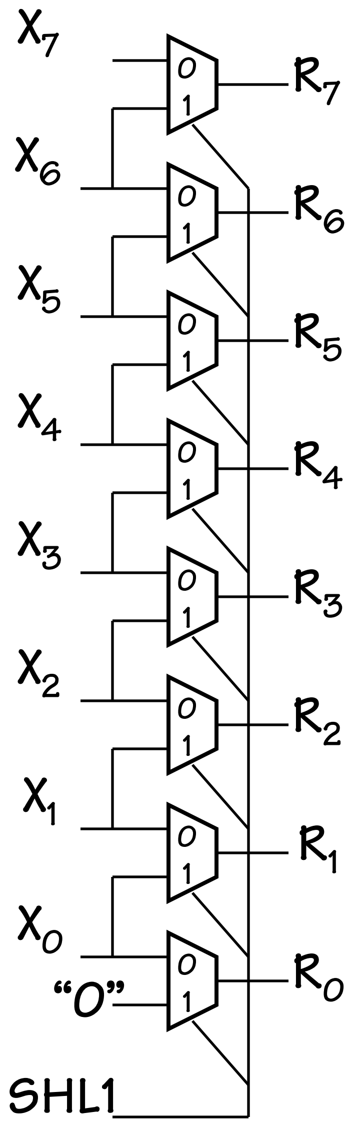
$(X << 1) = 00101000_2 = 40_{10}$

Right Shift:

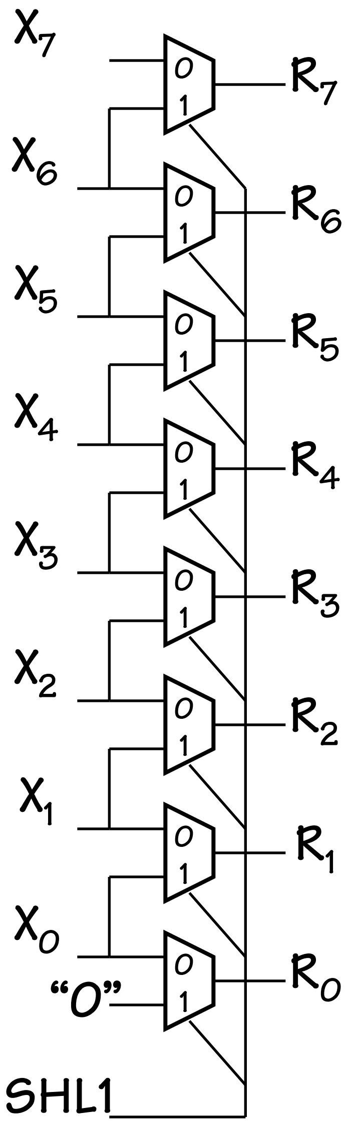
$(X >> 1) = 00001010_2 = 10_{10}$

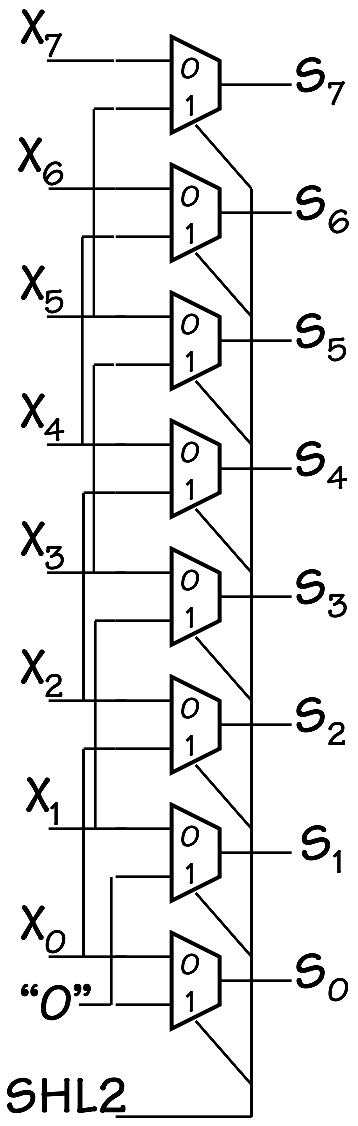
Signed or "Arithmetic" Right Shift:

$(-X >> 1) = (11101100_2 >> 1) = 11110110_2 = -10_{10}$



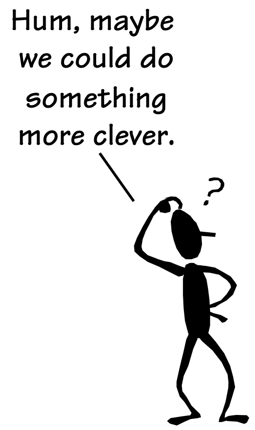
More Shifting

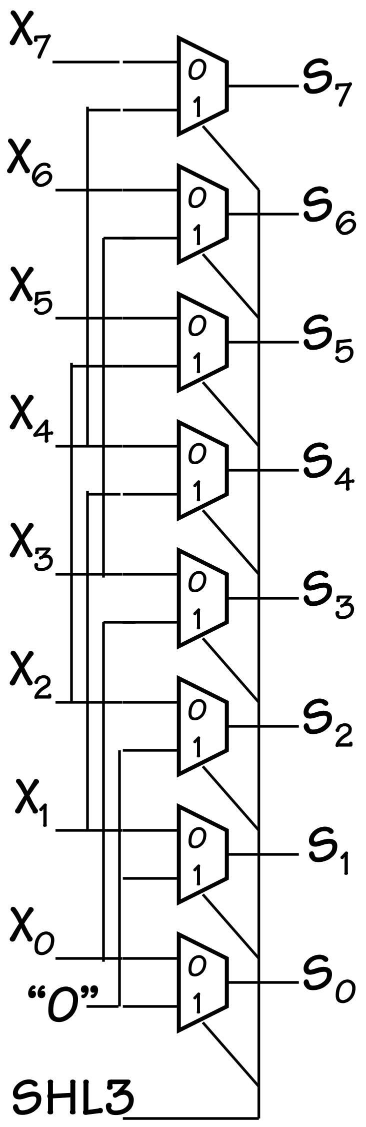




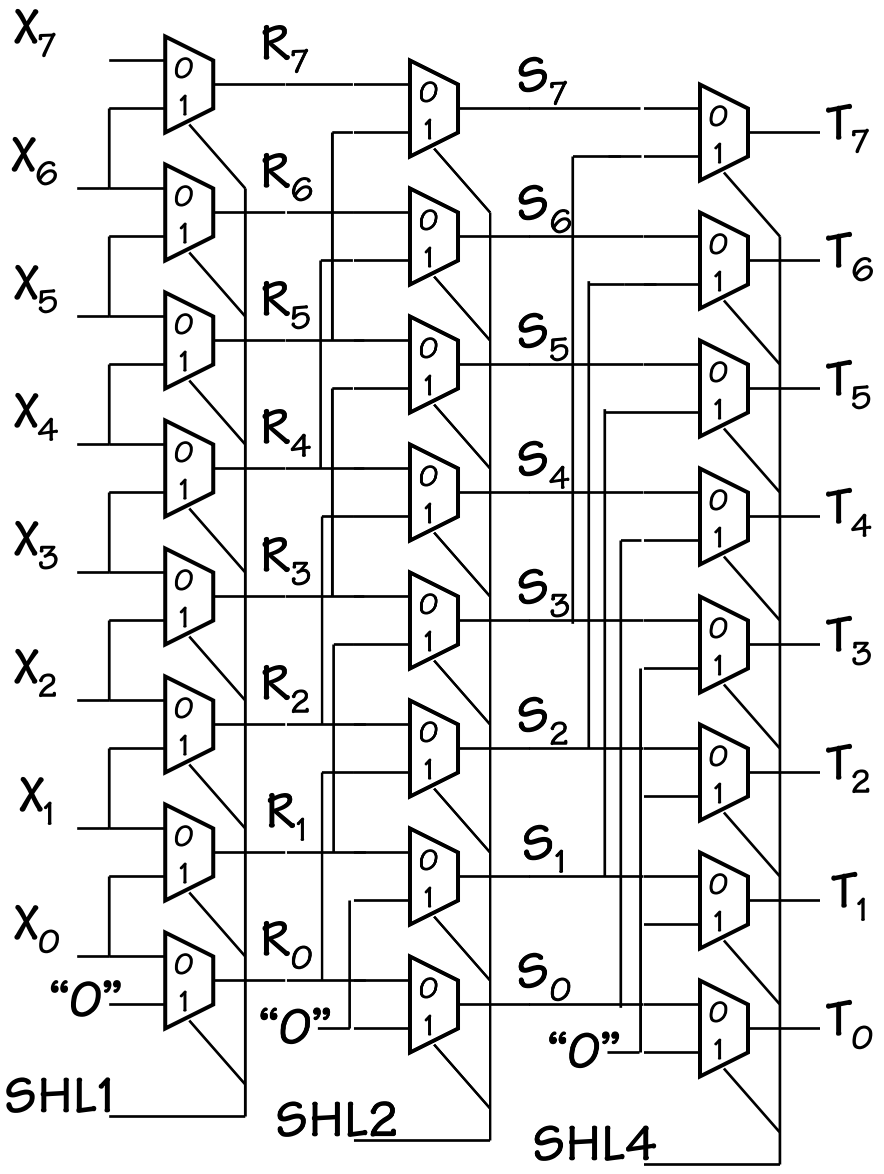
Using the same basic idea we can build left shifters of arbitrary sizes using muxes.

Each shift amount requires its own set of muxes.





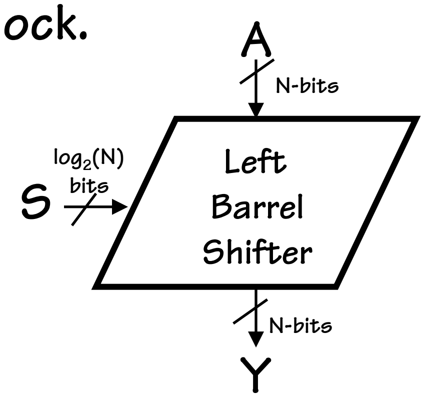
Barrel Shifting



If we connect our "shift-left -two" shifter to the output of our "shift-left-one" we can shift by 0, 1, 2, or 3 bits.

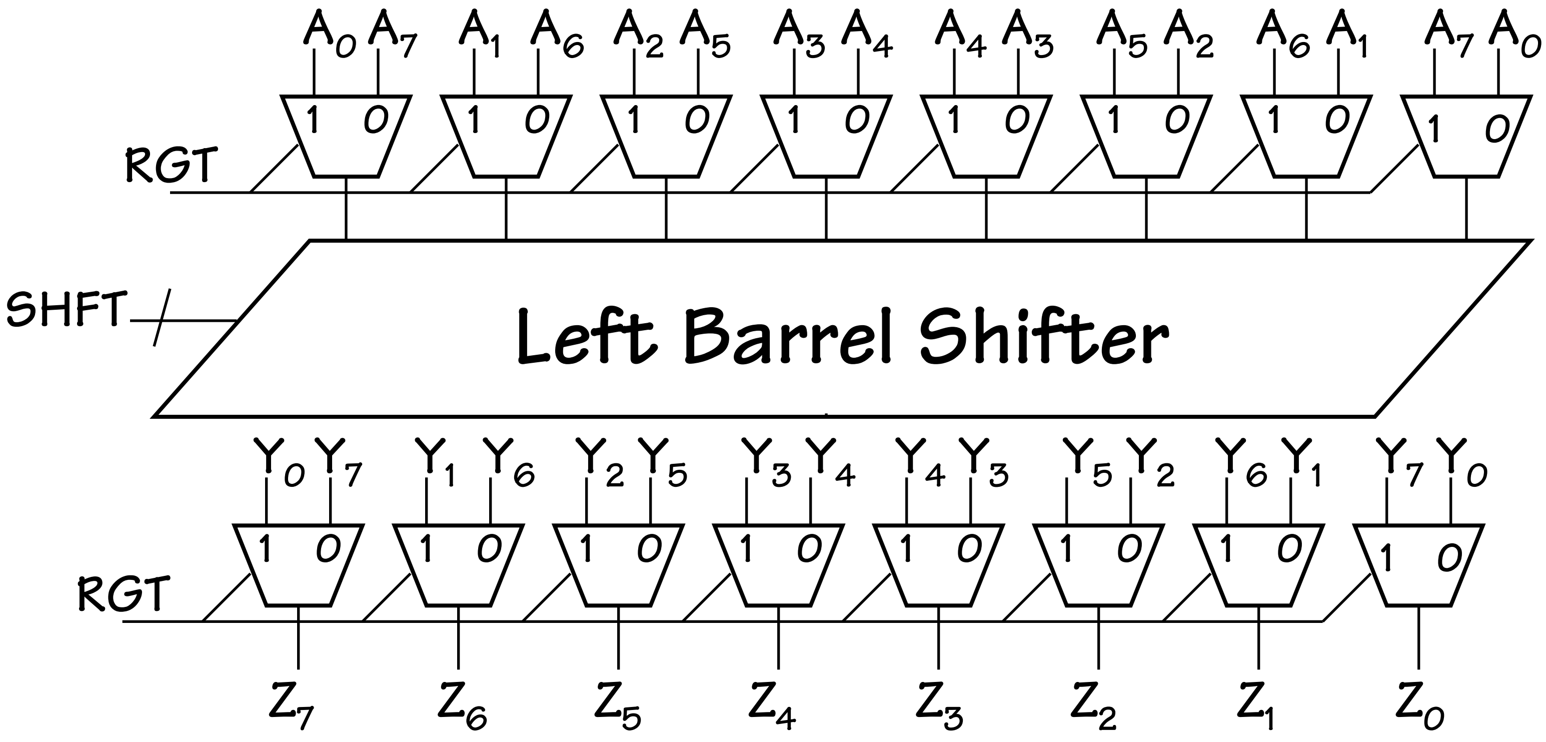
And, if we add one more "shift-left-4" shifter we can do any shift up to 7 bits!

So, let's put a box around it and call it a new functional block.



Barrel Shifting with a Twist

At this point it would be straightforward to construct a "Right barrel shifter" unit. However, a simple trick that enables a left shifter to do both.



Boolean Operations

We also need to perform logical operations on groups of bits.
Which ones?

ANDing is useful for “masking” off groups of bits.
ex. 10101110 & 00001111 = 00001110 (mask selects last 4 bits)

ANDing is also useful for “clearing” groups of bits.
ex. 10101110 & 00001111 = 00001110 (0’s clear first 4 bits)

ORing is useful for “setting” groups of bits.
ex. 10101110 | 00001111 = 10101111 (1’s set last 4 bits)

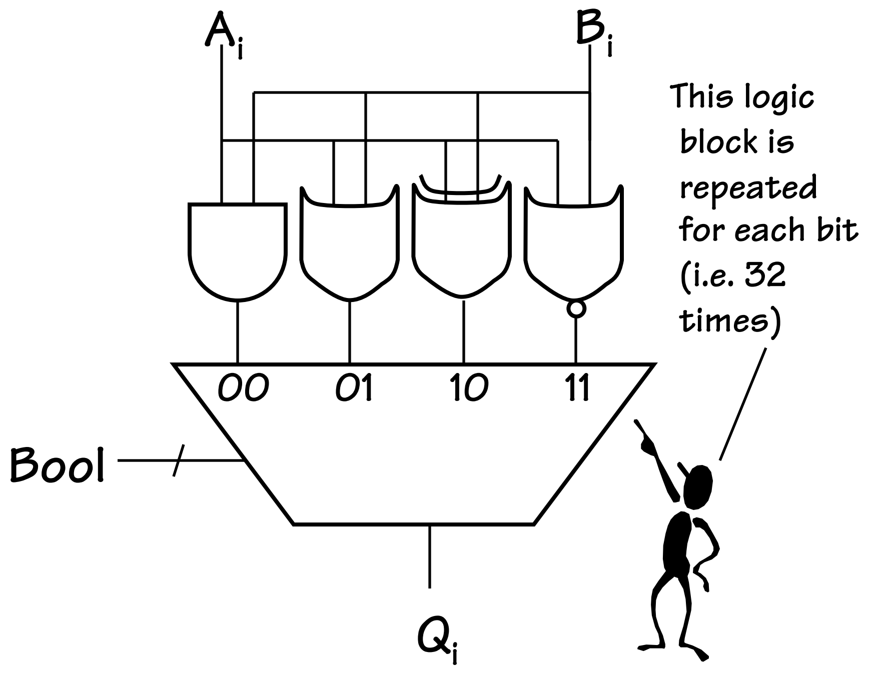
XORing is useful for “complementing” groups of bits.
ex. 10101110 ^ 00001111 = 10100001 (1’s complement last 4 bits)

NORing is useful.. Uhm, because John Hennessy says it is!
ex. ~(10101110 | 00001111) = 01010000 (0’s complement, 1’s clear)

Boolean Unit (The book's way)

It is simple to build up a Boolean unit using primitive gates and a mux to select the function.

Since there is no interconnection between bits, this unit can be simply replicated at each position. The cost is about 7 gates per bit. One for each primitive function, and approx 3 for the 4-input mux.

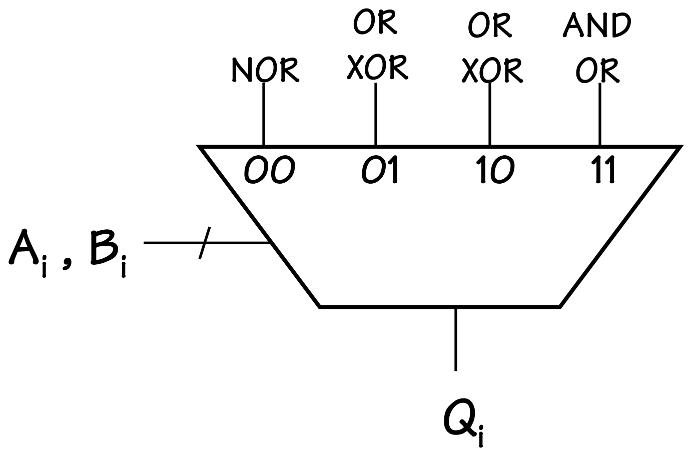


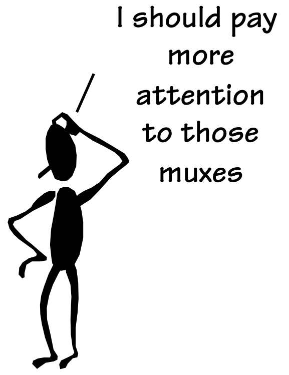
This is a straightforward, but not too elegant of a design.

Cooler Bools

We can better leverage a mux's capabilities in our Boolean unit design, by connecting the bits to the select lines.

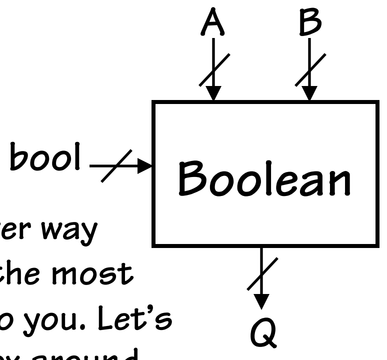
Why is this better?





1) While it might take a little logic to decode the truth table inputs, you only have to do it once, independent of the number of bits.

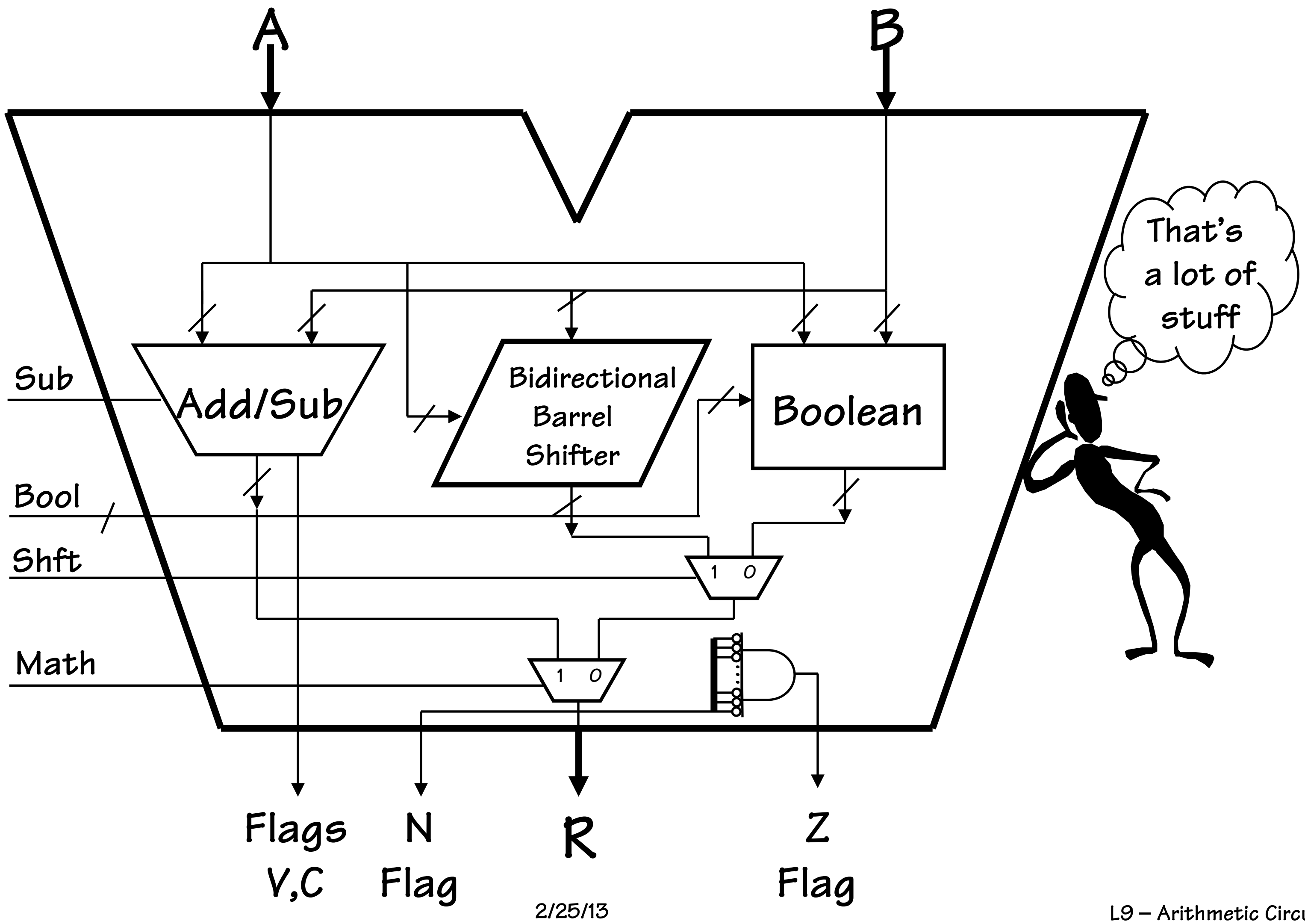
2) It is trivial to extend this module to support any 2-bit logical function. (How about NAND, John? Actually A & /B might be more useful)



Which ever way makes the most sense to you. Let's get a box around it!

An ALU, at Last

We give the "Math Center" of a computer a special name-- the Arithmetic Logic Unit. For us, it just a big box!


A
B
Sub
Bool
Shft
Math
Add/Sub
Bidirectional
Barrel
Shifter
Boolean
1
0
1
0
That's
a lot of
stuff
Flags
V,C
N
Flag
R
Z
Flag