Tree-Structured Indexes

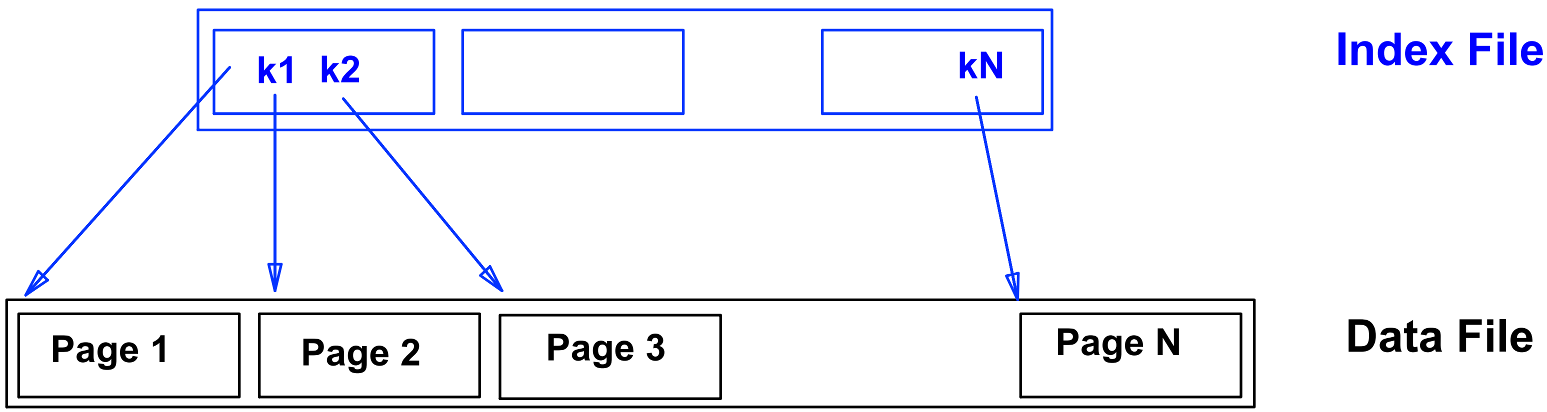
Chapter 10

Introduction

- *As for any index, 3 alternatives for data entries* **k***:
  - index refers to actual data record with key value **k**
  - index refers to list of <**k**, rid> pairs
  - index refers to list of <**k**, [rid list]>
- Choice is orthogonal to the *indexing technique* used to locate data entries **k***.
- Tree-structured indexing techniques support both *range searches* and *equality searches*.
- *ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.

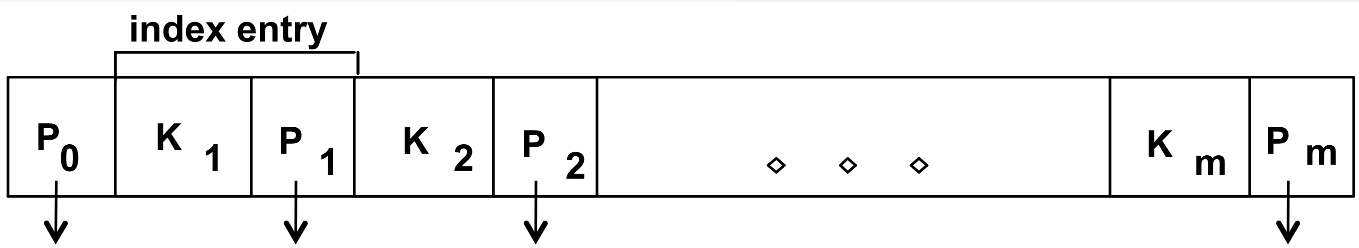
Range Searches

- *"Find all students with gpa > 3.0"*
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search can be quite high (must read entire page to access one record).
- Simple idea: Create an `index' file.

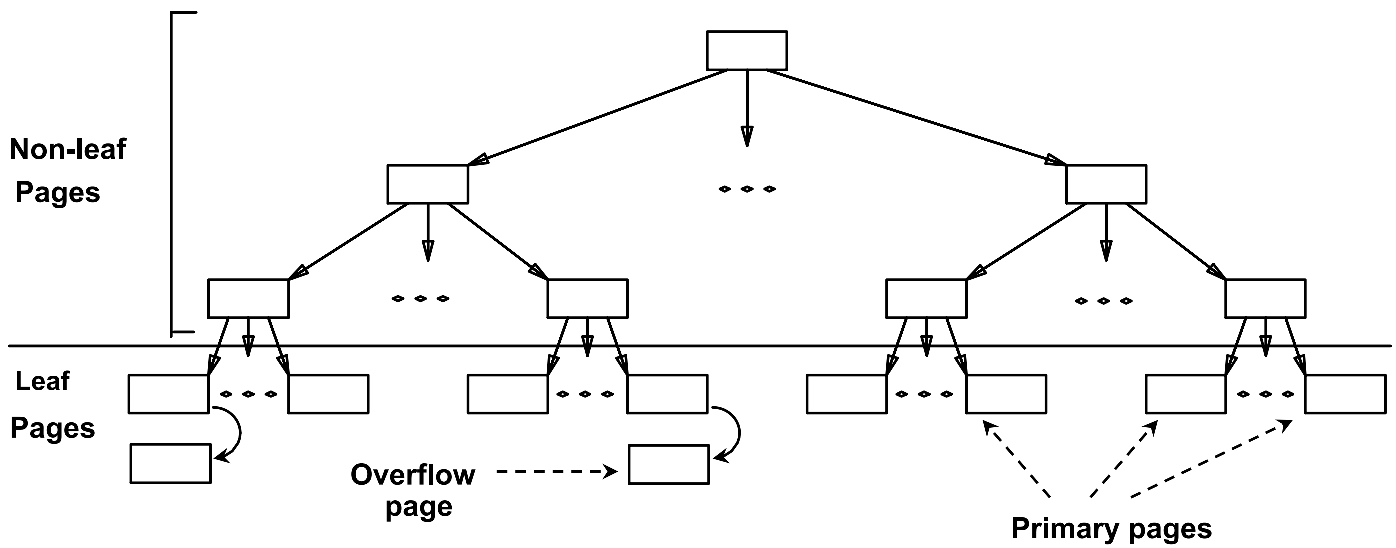


☛ *Can do binary search on (smaller) index file!*

ISAM – *Indexed Sequential Access Method*



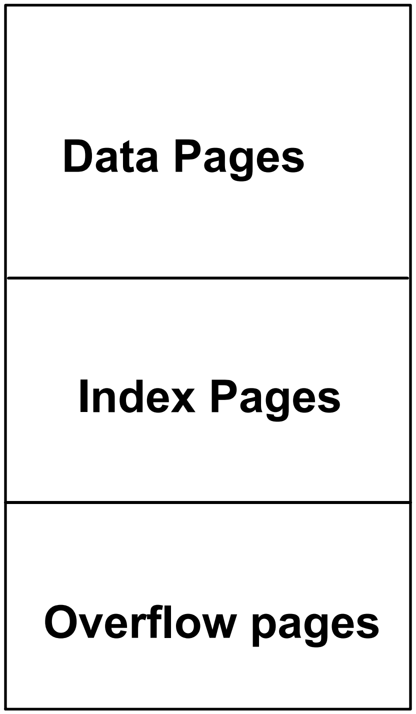
- Index file may be quite large.
- Can be applied hierarchically!



☛ *Leaf pages contain data entries (i.e. actual records or <key, rid> pairs.*

Comments on ISAM

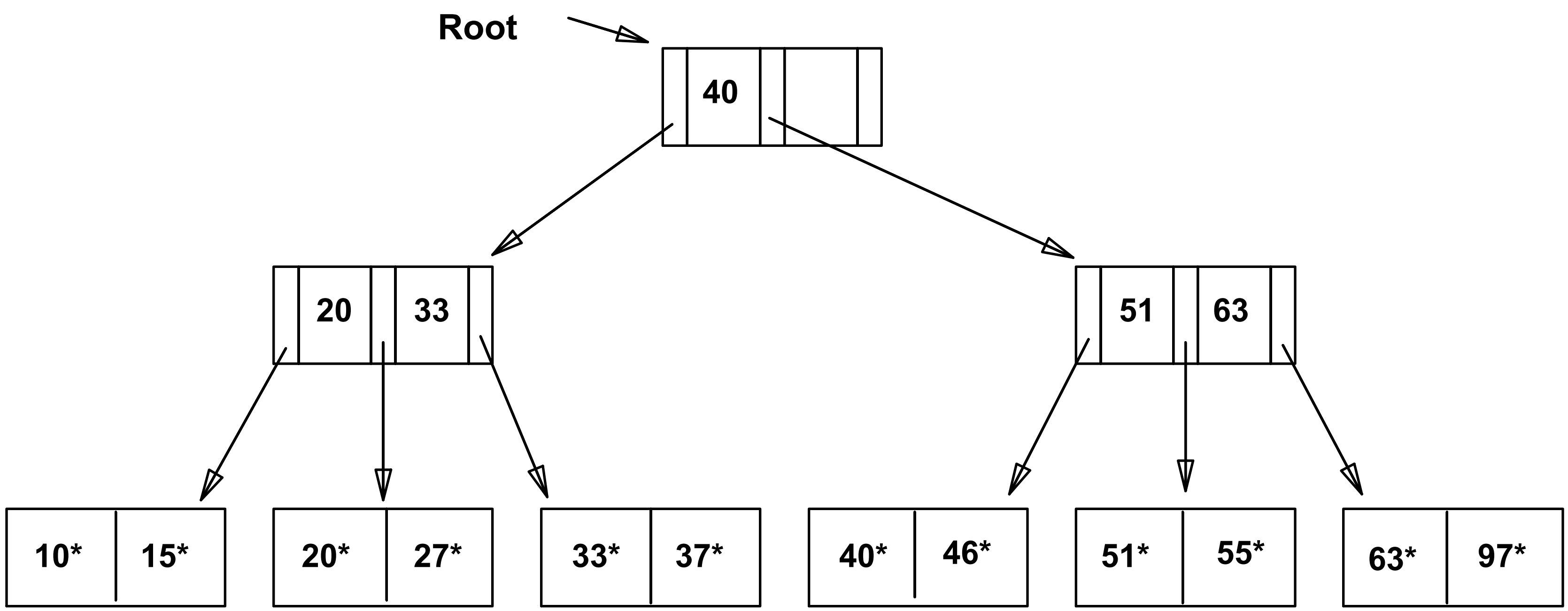
- *File creation*: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then space for overflow pages.
- *Index entries*: <search key value, page id>; they `direct' search for *data entries*, which are in leaf pages.
- *Search*: Start at root; use key comparisons to go to leaf. Cost $\log_F N$
  F = # entries/index pg, N = # leaf pgs
- *Insert*: Find leaf data entry belongs to, put it there if space is available, else allocate an overflow page, put it there, and link it in.
- *Delete*: Find and remove from leaf; if empty overflow page, de-allocate.



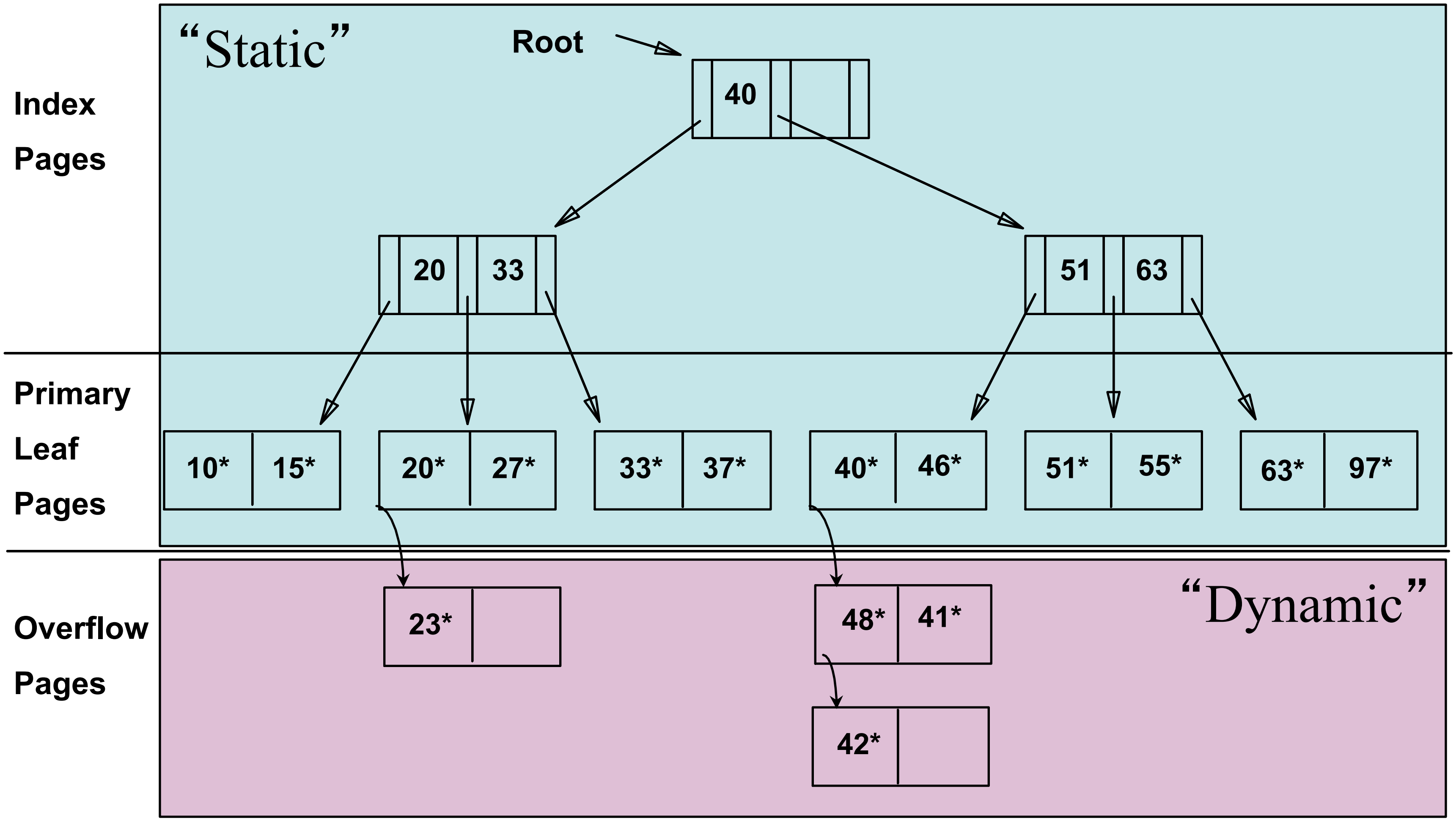
☛ **Static tree structure**: *inserts/deletes affect only leaf pages.*

Example ISAM Tree

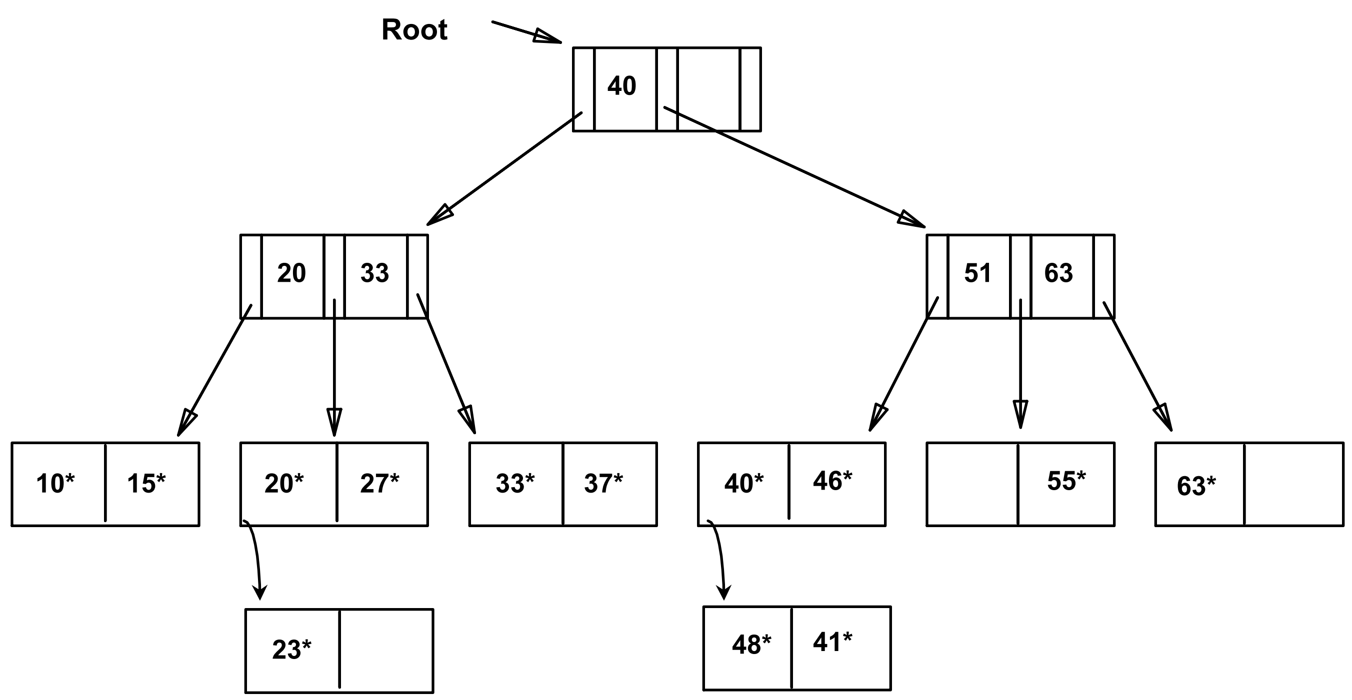
- Each node can hold 2 entries; no need for `next-leaf-page' pointers. (Why?)



After Inserting 23*, 48*, 41*, 42* ...



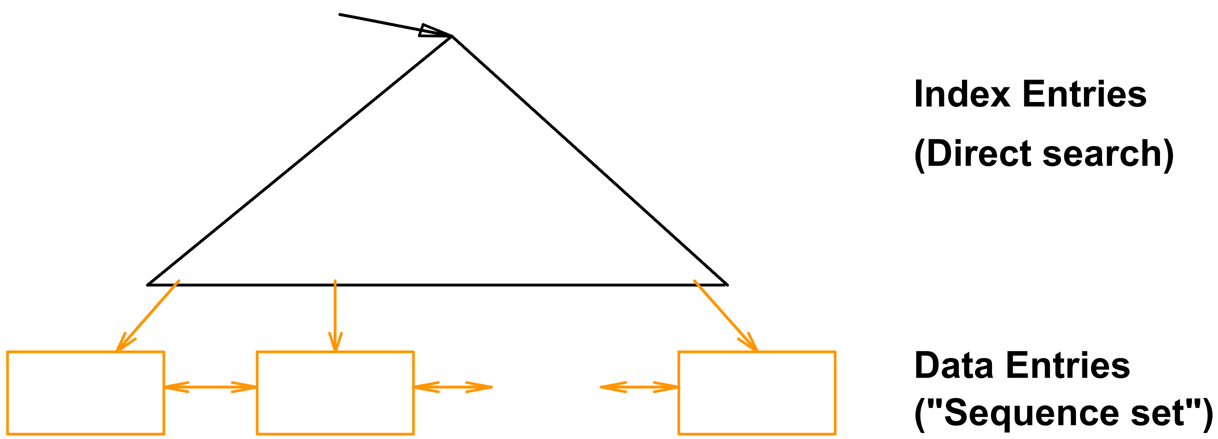
… Then Deleting 42*, 51*, 97*



☛ *Note that 51* appears in index, but not in leaf!*

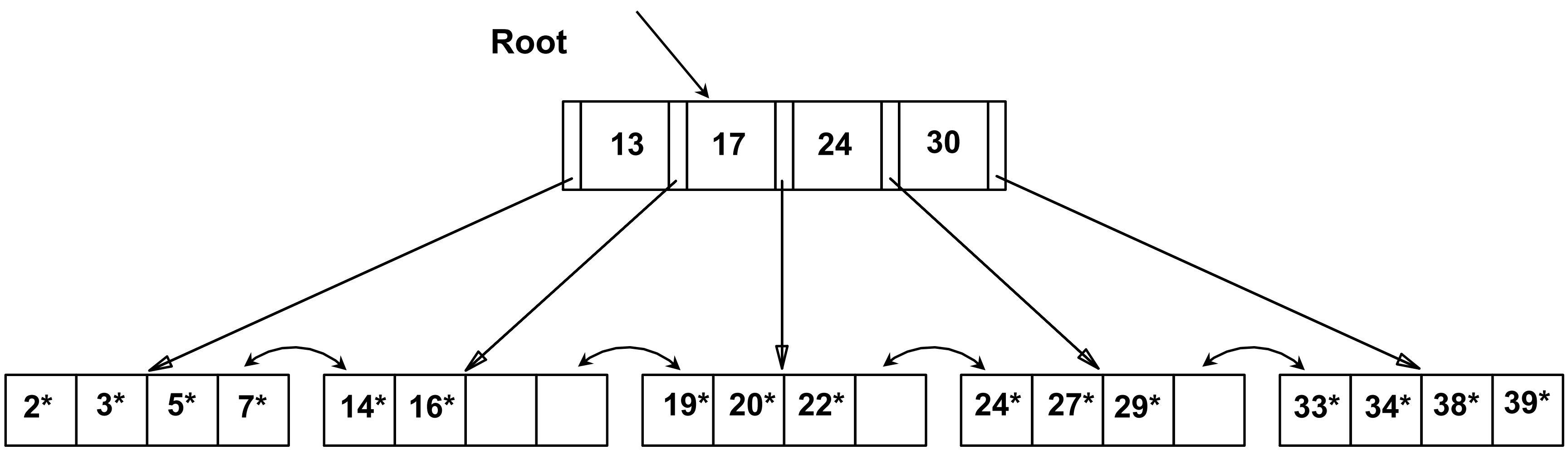
B+ Tree: Most Widely Used Index

- Insert/delete at $\log_F N$ cost; keep tree *balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy. Each internal non-root node contains **d** <= $\underline{m}$ <= 2**d** entries. The parameter **d** is called the *order* of the tree.
- Supports equality and range-searches efficiently.



Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf (as in ISAM).
- Search for 5*, 15*, all data entries >= 24* ...



☛ *Based on the search for 15*, we* *<u>know</u>* *it is not in the tree!*

B+ Trees in Practice

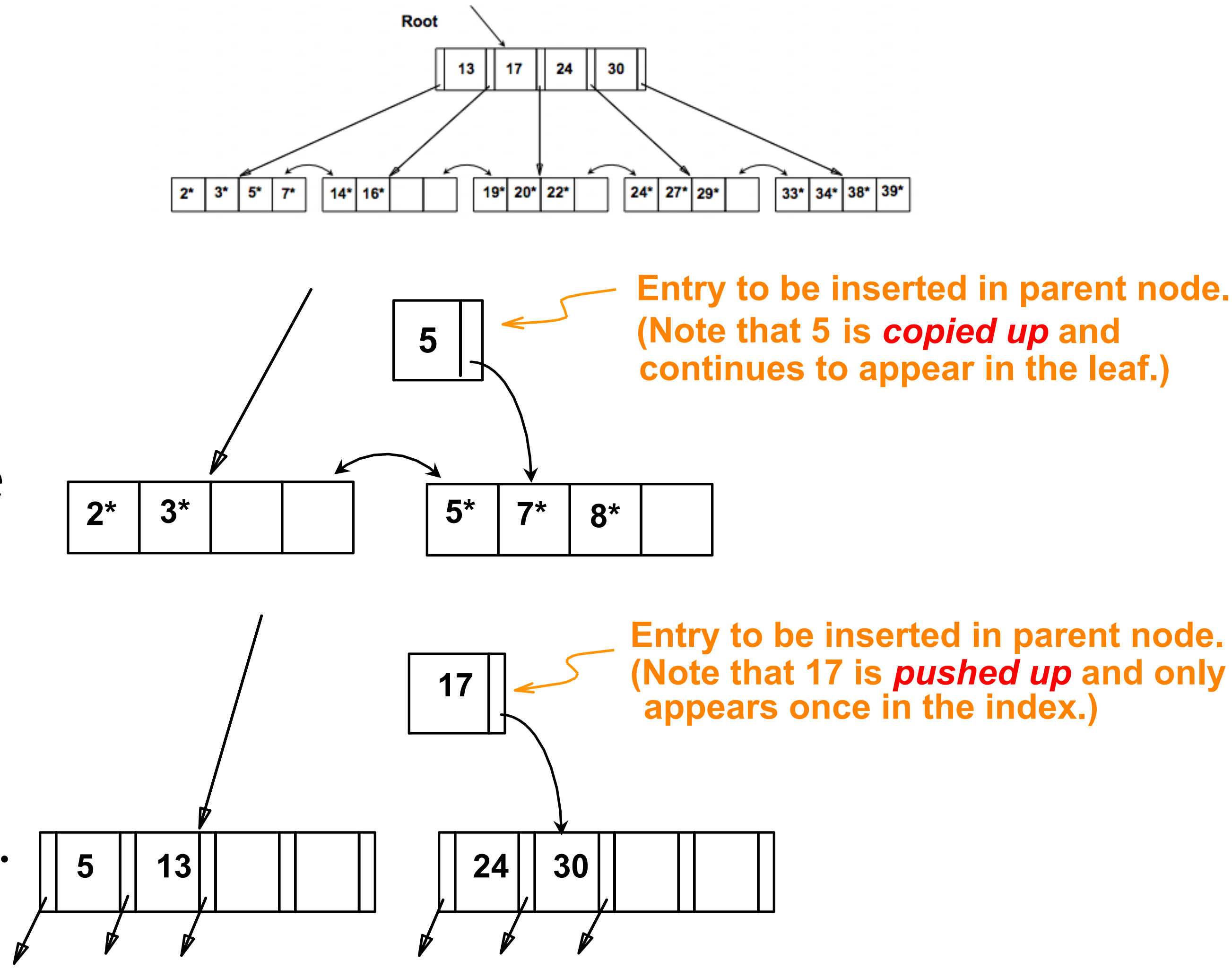
- Typical order: 100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 Mbytes

Inserting into a B+ Tree

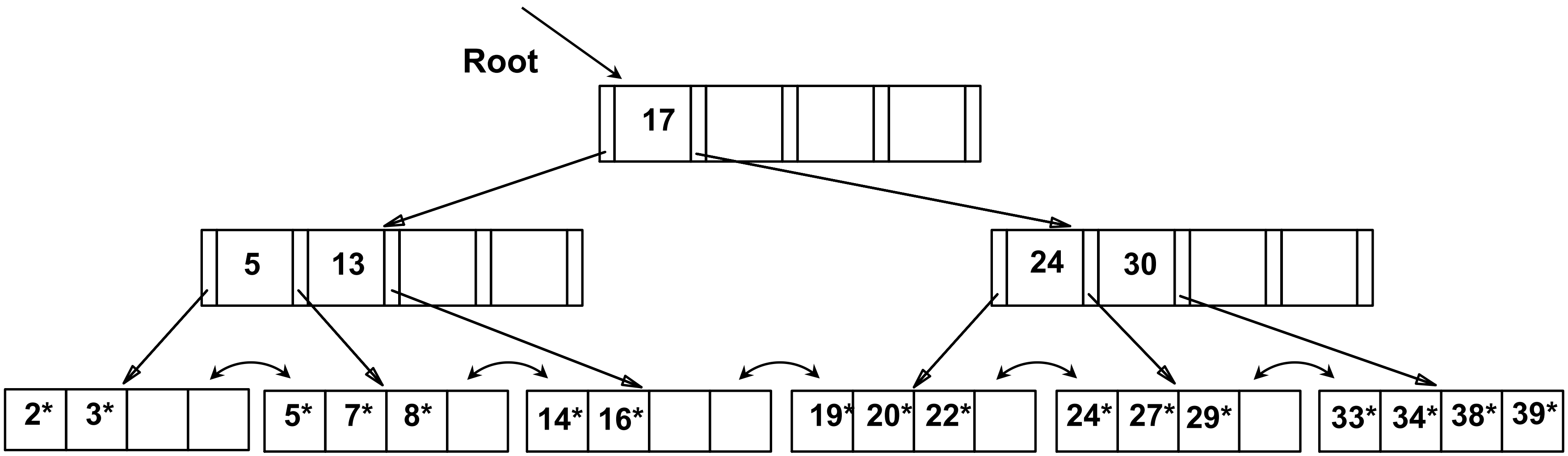
- Find correct leaf *L*.
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Allocate new node
    - Redistribute entries evenly
    - **Copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

Inserting 8 into Example B+ Tree*

- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.
- Note difference between *copy-up* and *push-up;* be sure you understand the reasons for this.

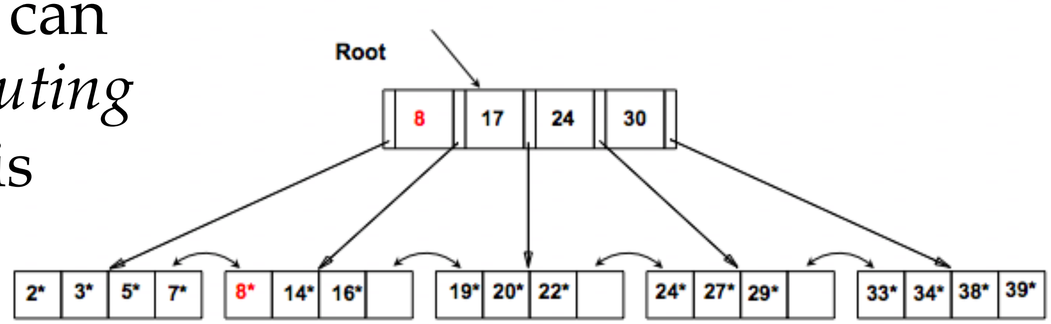


*Example B+ Tree After Inserting 8**



❖ Notice that root was split, leading to increase in height.

❖ In this example, we can avoid split by *redistributing* entries; however, this is usually not done in practice.

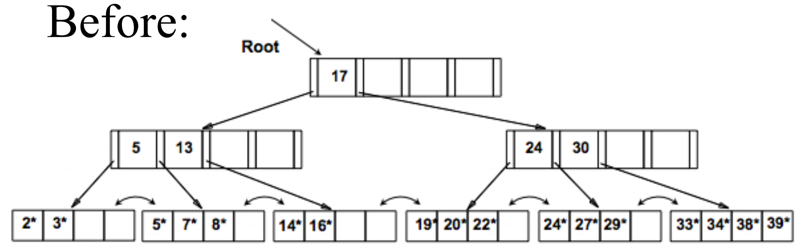


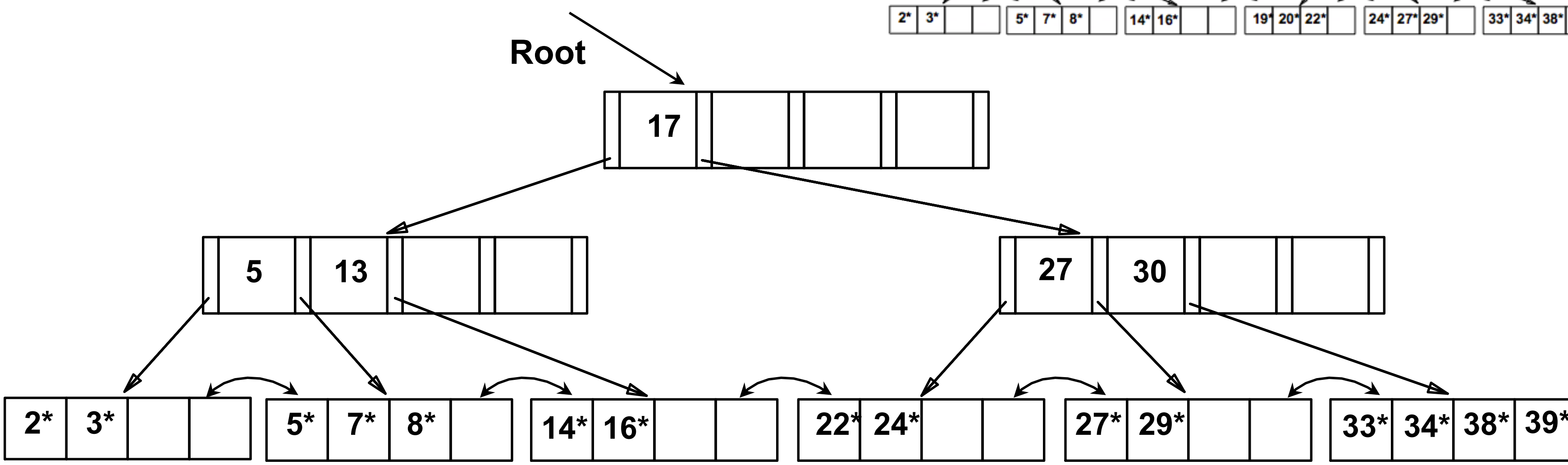
Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* with entry, if it exists.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing keys from *sibling* *(adjacent node with same parent as L)*.
    - If redistribution fails, *merge* *L* and sibling.
- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

Example Tree After (Inserting 8, Then) Deleting 19* and 20* ...*

Before:

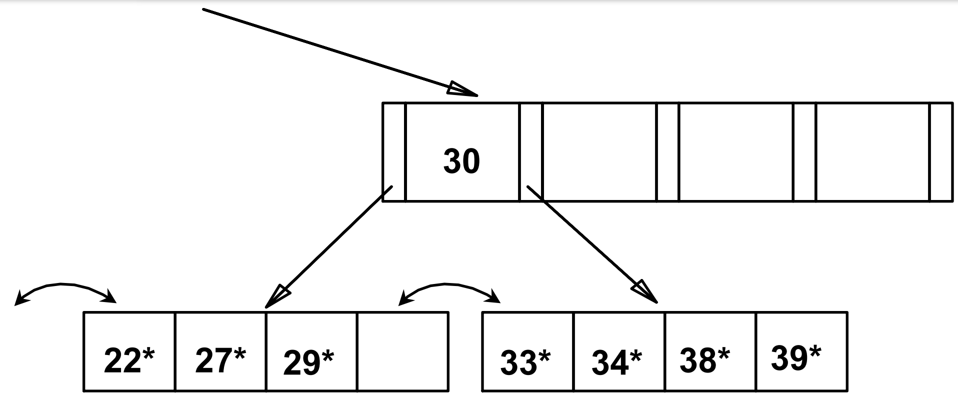


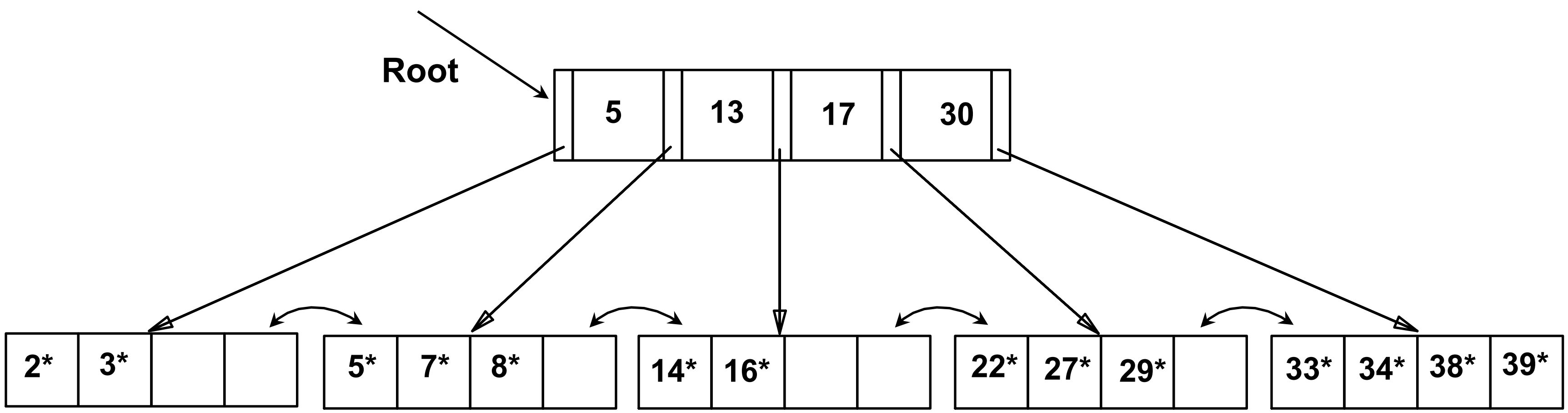


- Deleting 19* is easy.
- Deleting 20* is done with redistribution. Notice how middle key is *copied up*.

… *And Then Deleting 24**

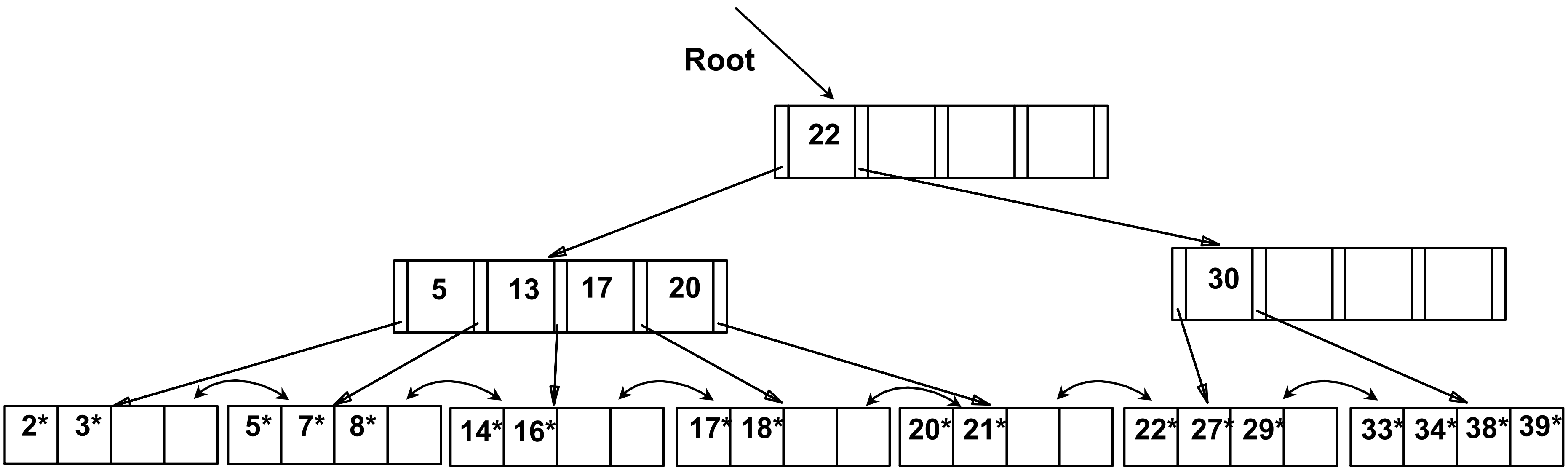
- Must merge.
- Observe `*toss*' of index entry (on right), and `*pull down*' of index entry (below).





Example of Non-leaf Redistribution

- Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can redistribute entry from left child of root to right child.



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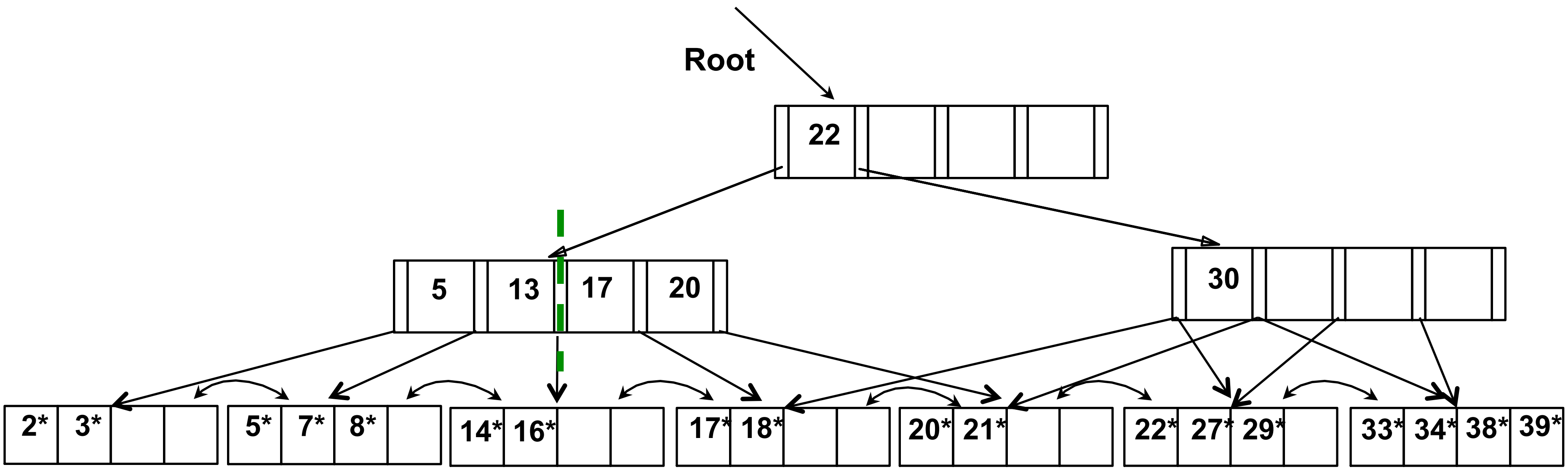
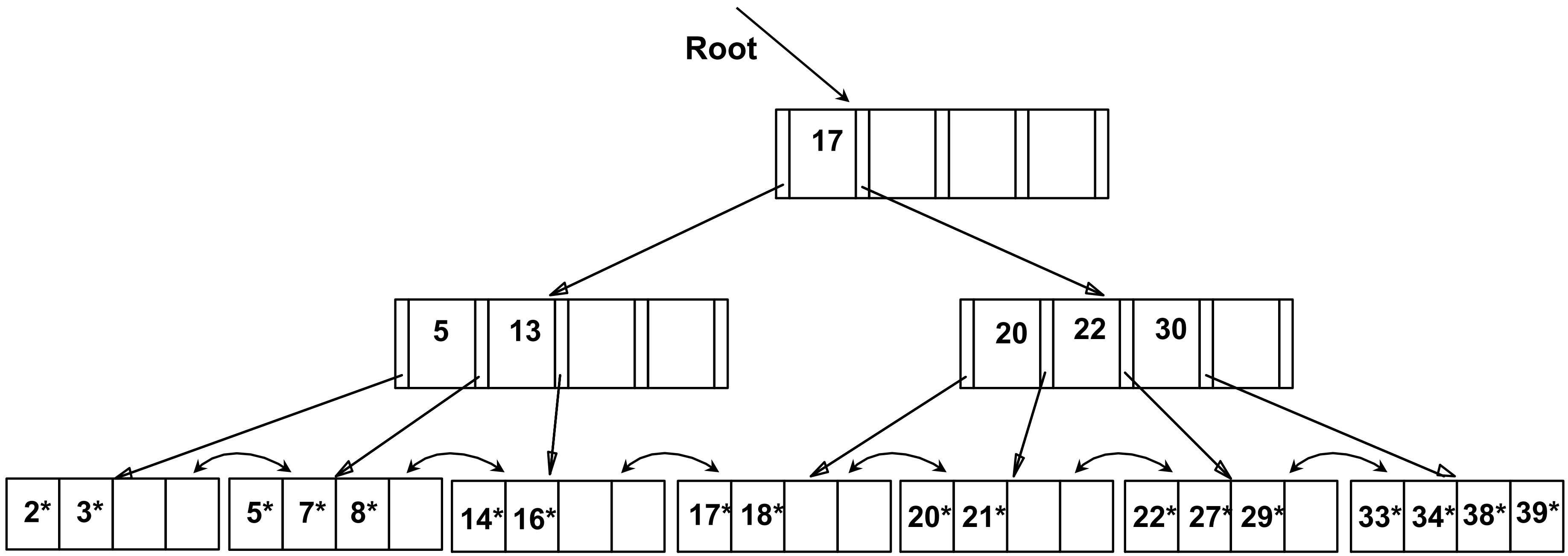


After Redistribution

- Intuitively, entries are redistributed by '*pushing through*' the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.

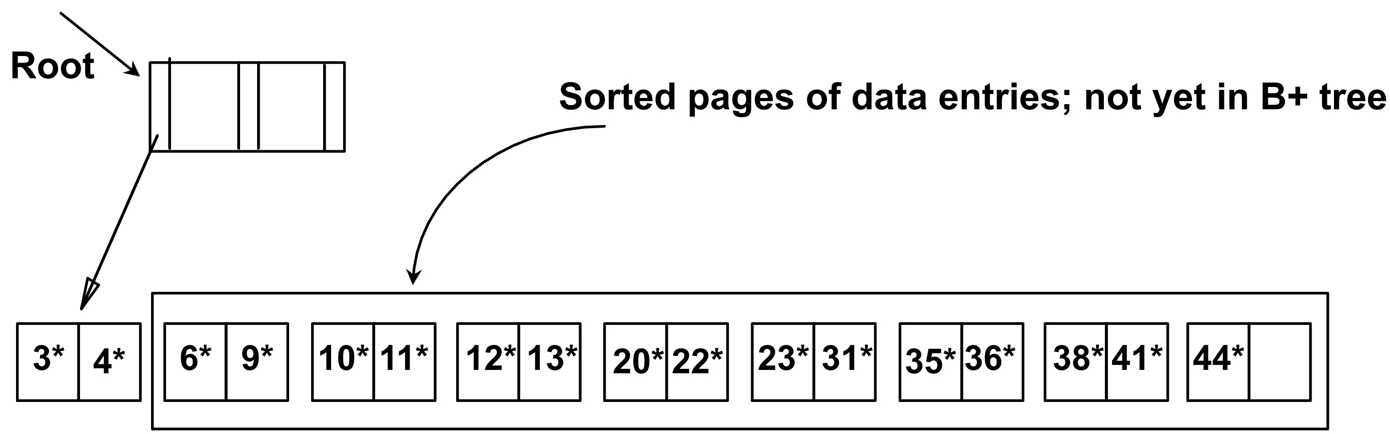


Prefix Key Compression

- Important to increase fan-out. (Why?)
- Key values in index entries only "direct traffic"; can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete must be suitably modified.

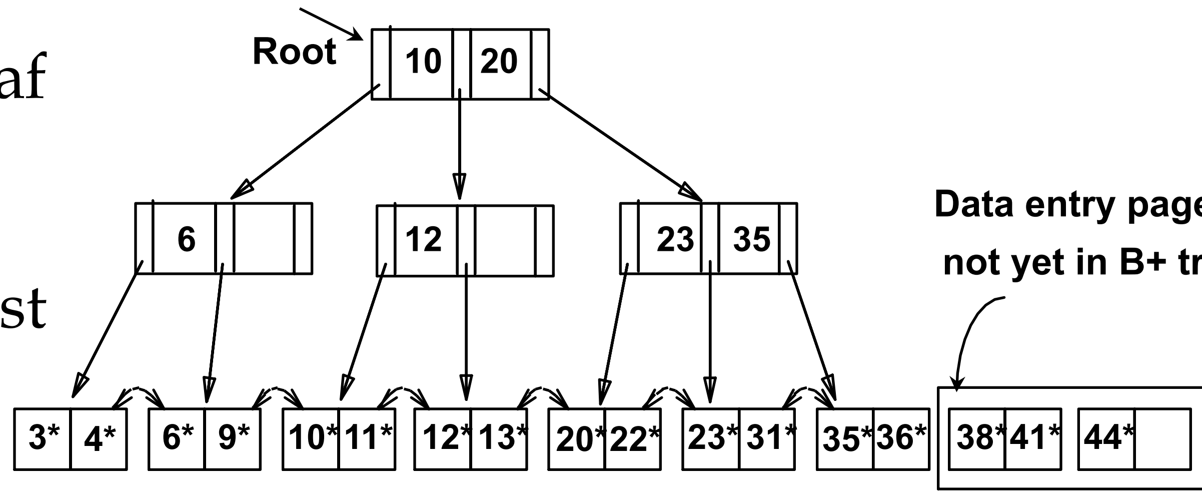
Bulk Loading of a B+ Tree

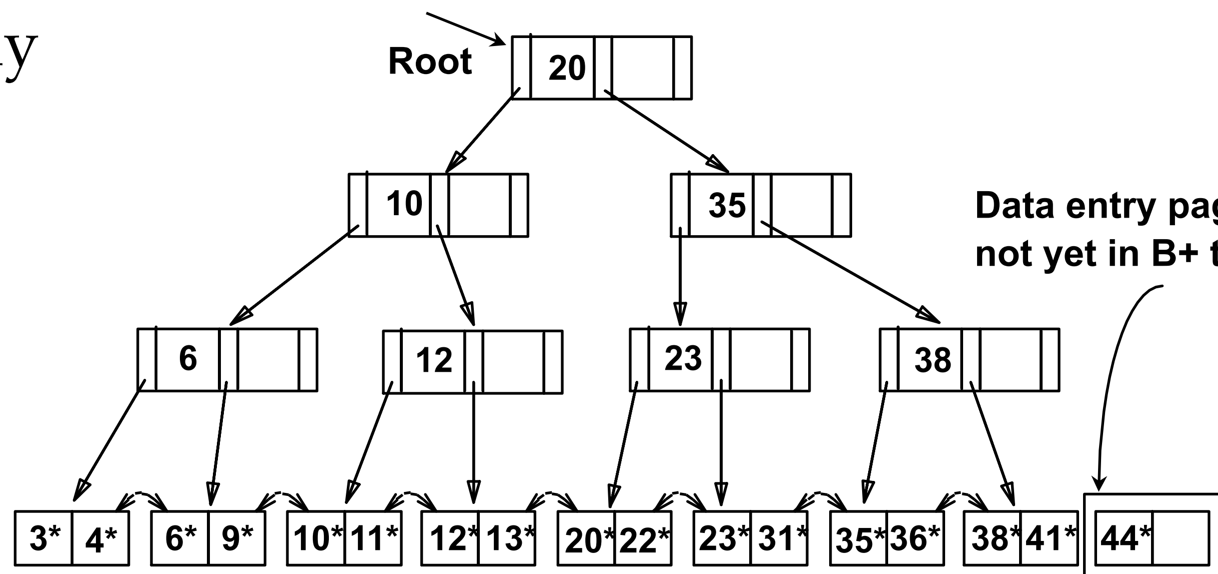
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- *Bulk Loading* can be done much more efficiently.
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially if one considers locking!





Summary of Bulk Loading

- Option 1: multiple inserts.
  - Slow.
  - Does not give sequential storage of leaves.
- Option 2: *Bulk Loading*
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control "fill factor" on pages.

A Note on "Order"

- *Order* (**d**) concept replaced by physical space criterion in practice (`*at least half-full*').
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean differnt nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.
- ISAM is a static structure.
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

Summary (Contd.)

- Typically, 67% occupancy on average.
- Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
- If data entries are data records, splits can change rids!

- Key compression increases fanout, reduces height.
- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.