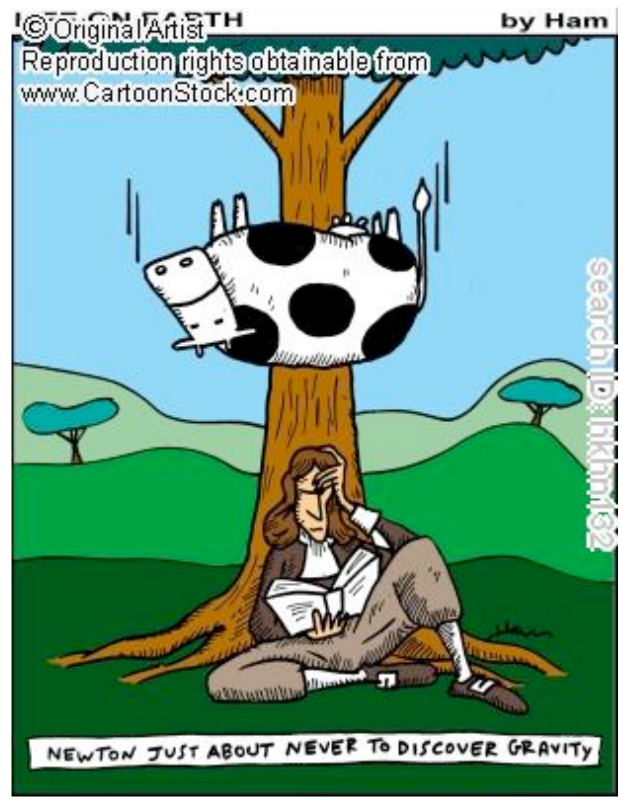
# *Relational Calculus*

Chapter 4.3-4.5



# *Relational Calculus*

- Comes in two flavors: *Tuple relational calculus* (TRC) and *Domain relational calculus* (DRC).
- Calculus has *variables, constants, comparison ops, logical connectives* and *quantifiers*.
  - *TRC*: Variables range over (i.e., get bound to) *tuples*.
  - *DRC*: Variables range over *domain elements* (= field values).
  - Both TRC and DRC are simple subsets of first-order logic.
- Expressions in the calculus are called *formulas with unbound formal variables*. An answer tuple is essentially an assignment of constants to these variables that make the formula evaluate to *true*.

# *A Fork in the Road*

- TRC and DRC are semantically similar
- In TRC, tuples share an equal status as variables, and field referencing can be used to select tuple parts
- In DRC, formal variables are explicit
- In the book you will find extensive discussions and examples of TRC Queries (Sections 4.3.1) and a lesser treatment of DRC.
- To even things out, in this lecture I will focus on DRC examples

# *Domain Relational Calculus*

- *Query* has the form:

$$\left\{\left\langle x1, x2, \ldots, xn \right\rangle \mid p\left(\left\langle x1, x2, \ldots, xn \right\rangle\right)\right\}$$

- *Answer* includes all tuples $\left\langle x1, x2, \ldots, xn \right\rangle$ that make the *formula* $p\left(\left\langle x1, x2, \ldots, xn \right\rangle\right)$ be *true*.
- *Formula* is recursively defined, starting with simple *atomic formulas* (getting tuples from relations or making comparisons of values), and building bigger and better formulas using the *logical connectives*.

# DRC Formulas

- *Atomic formula:*
  - $\langle x1, x2, \ldots, xn \rangle \in Rname$, or X *op* Y, or X *op* constant
  - *op* is one of $<, >, =, \leq, \geq, \neq$
- *Formula:*
  - an atomic formula, or
  - $\neg p, p \wedge q, p \vee q$, where p and q are formulas, or
  - $\exists X(p(X))$, where variable X is *free* in p(X), or
  - $\forall X(p(X))$, where variable X is *free* in p(X)

$\exists$X(p(X)) is read as "*there exists a setting of the variable X such that p(X) is true*". $\forall$X(p(X)) is read as "*for all values of X, p(X) is true*"

# *Free and Bound Variables*

- The use of quantifiers $\exists X$ and $\forall X$ in a formula is said to *bind* X.
  - A variable that is not bound is free.
- Let us revisit the definition of a query:

$$\left\{\left\langle x1, x2, \ldots, xn \right\rangle \mid p\left(\left\langle x1, x2, \ldots, xn \right\rangle\right)\right\}$$

- There is an important restriction: the variables x1, ..., xn that appear to the left of '|' must be the *only* free variables in the formula p(...).

# Examples

- Recall the example relations from last lecture

Sailors:

| sid | sname | rating | age |
|---|---|---|---|
| 22 | Dustin | 7 | 45.0 |
| 29 | Brutus | 1 | 33.0 |
| 31 | Lubber | 8 | 55.5 |
| 32 | Andy | 8 | 25.5 |
| 58 | Rusty | 10 | 35.0 |
| 64 | Horatio | 7 | 35.0 |
| 71 | Zorba | 10 | 16.0 |
| 74 | Horatio | 9 | 35.0 |
| 85 | Art | 3 | 25.5 |
| 95 | Bob | 3 | 63.5 |

Reservations:

| sid | bid | day |
|---|---|---|
| 22 | 101 | 10/10/98 |
| 22 | 102 | 10/10/98 |
| 22 | 103 | 10/8/98 |
| 22 | 104 | 10/7/98 |
| 31 | 102 | 11/10/98 |
| 31 | 103 | 11/6/98 |
| 31 | 104 | 11/12/98 |
| 64 | 101 | 9/5/98 |
| 64 | 102 | 9/8/98 |
| 74 | 103 | 9/8/98 |

Boats:

| bid | bname | color |
|---|---|---|
| 101 | Interlake | blue |
| 102 | Interlake | red |
| 103 | Clipper | green |
| 104 | Marine | red |

# *Find sailors with ratings > 7*

$$\left\{ \langle I,N,T,A \rangle \mid \langle I,N,T,A \rangle \in Sailors \wedge T > 7 \right\}$$

- The condition $\langle I,N,T,A \rangle \in Sailors$ ensures that the domain variables *I, N, T* and *A* are bound to fields of the same Sailors tuple.
- The term $\langle I,N,T,A \rangle$ to the left of `|' (which should be read as *such that*) says that every tuple $\langle I,N,T,A \rangle$ that satisfies $T > 7$ is in the answer.
- Modify this query to answer:
  - Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.

# *Same query using TRC*

- Find all sailors with ratings above 7

$$\{S \mid S \in Sailors \wedge S.rating > 7\}$$

- Note, here $S$ is a tuple variable

$$\{X \mid S \in Sailors(S.rating > 7 \wedge X.name = S.sname \wedge X.age = S.age)\}$$

- Here X is a tuple with 2 fields (name, age). This query implicitly specifies projection (π) and renaming (ρ) relational algebra operators

## *Find sailors rated > 7 who have reserved boat #103*

$$\left\{ \left\langle I,N,T,A \right\rangle \mid \left\langle I,N,T,A \right\rangle \in Sailors \wedge T > 7 \wedge \right.$$

$$\left. \exists\, Ir,Br,D \left( \left\langle Ir,Br,D \right\rangle \in \mathrm{Re}serves \wedge Ir = I \wedge Br = 103 \right) \right\}$$

- We have used $\exists\, Ir,\ Br,\ D\ \left(\ldots\right)$ as a shorthand for $\exists\, Ir \left( \exists\, Br \left( \exists\, D \left( \ldots \right) \right) \right)$
- Note the use of $\exists$ to find a tuple in Reserves that 'joins with' ($\bowtie$) the Sailors tuple under consideration.

# *Find sailors rated > 7 who've reserved a red boat*

$$\left\{ \langle I,N,T,A \rangle \mid \langle I,N,T,A \rangle \in Sailors \wedge T > 7 \wedge \right.$$
$$\exists\, Ir,Br,D \left( \langle Ir,Br,D \rangle \in Reserves \wedge Ir = I \wedge \right.$$
$$\left. \left. \exists\, B,BN,C \left( \langle B,BN,C \rangle \in Boats \wedge B = Br \wedge C = 'red' \right) \right) \right\}$$

- Observe how the parentheses control the scope of each quantifier's binding.
- This may look cumbersome, but with a good user interface, it is very intuitive. (MS Access, QBE)

# *Names of all Sailors who have reserved boat 103*

$$\{\langle N\rangle \mid \exists I,T,A(\langle I,N,T,A\rangle \in Sailor)$$

$$\wedge\, \exists Ir,Br,D(\langle Ir,Br,D\rangle \in Reserves \wedge Ir = I \wedge Br = 103)\}$$

- Note that only the *sname* field is retained in the answer and that only *N* is a free variable.
- A more compact version

$$\{\langle N\rangle \mid \exists I,T,A(\langle I,N,T,A\rangle \in Sailor)$$

$$\wedge\, \exists D(\langle I,103,D\rangle \in Reserves)\}$$

# *Names of Sailors who have reserved a boat named "Interlake"*

# *Sailors who've reserved all boats*

- Recall how queries of this type used of the "division" operator in relational algebra
- The trick is that we use "forall" quantification ($\forall$) in place of "there exists" quantification ($\exists$)
- Domains of variables are determined when they are bound
- Think of it as considering each variable's "domain" of independently in our substitution

| bid | bname | color |
|---|---|---|
| 101 | Interlake | blue |
| 101 | Interlake | red |
| 101 | Interlake | green |
| 101 | Clipper | blue |
| 101 | Clipper | red |
| 101 | Clipper | green |
| 101 | Marine | blue |
| 101 | Marine | red |
| 101 | Marine | green |
| 102 | Interlake | blue |
| | . . . | |
| 104 | Marine | green |
| 104 | marine | red |

# Sailors who've reserved all boats

$$\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge$$
$$\forall\ B,BN,C\ \big(\neg\big(\langle B,BN,C\rangle \in Boats\big)\ \vee$$
$$\big(\exists\ Ir,Br,D\ \big(\langle Ir,Br,D\rangle \in \mathrm{Re}serves \wedge I = Ir \wedge Br = B\big)\big)\big)\}$$

- Find all sailors $I$ such that for each 3-tuple $\langle B,BN,C\rangle$ either *it is not a tuple in Boats* or *there is a tuple in Reserves showing that sailor **I** has reserved it*.

# *Find sailors who've reserved all boats (again!)*

$$\Big\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge$$
$$\forall \langle B,BN,C\rangle \in Boats$$
$$\Big(\exists \langle Ir,Br,D\rangle \in \mathrm{Re}serves\big(I = Ir \wedge Br = B\big)\Big)\Big\}$$

- Simpler notation, same query. (Much clearer!)
- To find sailors who've reserved all red boats:

$$\ldots\ldots \quad \Big(C \neq 'red' \vee \exists \langle Ir,Br,D\rangle \in \mathrm{Re}serves\big(I = Ir \wedge Br = B\big)\Big)\Big\}$$

# *Unsafe Queries, Expressive Power*

- It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called *unsafe*.
  - e.g., $$\left\{ < I,N,T,A > \;\middle|\; < I,N,T,A > \notin \; Sailors \right\}$$
- It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- *Relational Completeness*: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.

# Summary

- Relational calculus is non-operational, and users define queries in terms of what they want, not in terms of how to compute it. (Declarativeness.)
- Algebra and safe calculus have same expressive power, leading to the notion of relational completeness.