External Sorting

Problem Set #2 should be graded before Thursday

Problems Set #4 and Midterm results will be here when you get back from break
Why Sort?

- A classic problem in computer science!
- Advantages of requesting data in sorted order
  - gathers duplicates
  - allows for efficient searches
- Sorting is first step in bulk loading B+ tree index.
- Sort-merge join algorithm involves sorting.
- Problem: sort 20Gb of data with 1Gb of RAM.
  - why not let the OS handle it with virtual memory?
2-Way Sort: Requires 3 Buffers

- Pass 1: Read a page, sort it, write it.
  - only one buffer page is used
- Pass 2, 3, ..., N etc.:
  - Read two pages, merge them, and write merged page
  - Requires three buffer pages.
Two-Way External Merge Sort

- Each pass we read + write each page in file.
- $N$ pages in the file => the number of passes
  
  \[ \lceil \log_2 N \rceil + 1 \]

- So total cost is 
  (2N = N reads + N writes):

  \[ 2N \left( \lceil \log_2 N \rceil + 1 \right) \]

- Idea: Divide and conquer: sort pages and merge

Input file
PASS 0
1-page runs
PASS 1
2-page runs
PASS 2
4-page runs
PASS 3
8-page runs
General External Merge Sort

More than 3 buffer pages. How can we utilize them?

- Key Insight #1: We can merge more than 2 input buffers at a time... affects fanout \( \log \) base of log!
- Key Insight #2: The output buffer is generated incrementally, so only one buffer page is needed for any size of run!
- To sort a file with \( N \) pages using \( B \) buffer pages:
  - Pass 0: use \( B \) buffer pages. Produce \( \lceil N / B \rceil \) sorted runs of \( B \) pages each.
  - Pass 2, ..., etc.: merge \( B-1 \) runs, leaving one page for output.
General External Merge Sort

More than 3 buffer pages. How can we utilize them?

To sort a file with $N$ pages using $B$ buffer pages:
- Pass 0: use $B$ buffer pages. Produce $\lceil N / B \rceil$ sorted runs of $B$ pages each.
More than 3 buffer pages. How can we utilize them?

- To sort a file with $N$ pages using $B$ buffer pages:
  - Pass 0: use $B$ buffer pages. Produce $\lceil N / B \rceil$ sorted runs of $B$ pages each.
  - Pass 1, …, etc.: merge $B-1$ runs. Repeat.
Cost of External Merge Sort

- Number of passes: $1 + \lceil \log_{B-1} \lceil \frac{N}{B} \rceil \rceil$
- Cost = $2N \times (\# \text{ of passes})$
- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 0: $\lceil \frac{108}{5} \rceil = 22$ sorted runs of 5 pages each (last run is only 3 pages)
  - Pass 1: $\lceil \frac{22}{4} \rceil = 6$ sorted runs of 20 pages each (last run is only 8 pages)
  - Pass 2: $\lceil \frac{6}{4} \rceil = 2$ sorted runs, 80 pages and 28 pages
  - Pass 3: Sorted file of 108 pages
# Number External Sort Passes

<table>
<thead>
<tr>
<th>N</th>
<th>B=3</th>
<th>B=5</th>
<th>B=9</th>
<th>B=17</th>
<th>B=129</th>
<th>B=257</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1,000</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>10,000</td>
<td>13</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>100,000</td>
<td>17</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1,000,000</td>
<td>20</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1,000,000,000</td>
<td>30</td>
<td>15</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
Internal Sort Algorithm

- Quicksort is a fast way to sort in memory.
  - Very fast on average
  - Worse case $N^2$ (i.e. bad pivots)

- Alternatives
  - Heap Sort, stable and always $O(N \text{log} N)$
  - Merge Sort, same approach used in “out-of-core” sort but applied within a block recursively (low overhead)
    - Divides block into two halves, sorts each by dividing them recursively into two halves until there is only one item in the list. Then merges all of the "half-sized" lists while returning up the recursion.
  - Another Problem… waiting to fill the buffer pool
Sorting Stalls

- When a "top page" empties, we need to wait for it to be refilled
- While waiting, we can't fill the merge output buffer using the other top pages, because the next value merged might come from the next block of the exhausted run
Double Buffering

- To reduce wait time for I/O request to complete, can *prefetch* into a "shadow block".
- Potentially, more passes; in practice, most files *still* sorted in 2-3 passes.
Sorting Records!

- Sorting has become a blood sport!
  - Parallel external sorting is the name of the game ...

- 2015 FuxiSort (Alibaba Group, Inc.)
  - Sort 100Tbyte of 100 byte records
  - Typical DBMS: > 10 days
  - World record: 329 seconds
    - 2 Xeon E5-2630 2 2.3 GHz with 3134 nodes
    - Each node: 96 GB of RAM, and a 12x2 TB SATA disk

- New benchmarks proposed:
  - Minute Sort: How many can you sort in 1 minute?
  - Cloud Sort: How many $ per TB sorted?
Using B+ Trees for Sorting

- Scenario: Table to be sorted has B+ tree index on sorting column(s).
- Idea: Can retrieve records in order by traversing leaf pages.
- Is this a good idea?
- Cases to consider:
  - B+ tree is clustered  
    Good idea!
  - B+ tree is not clustered  
    Could be a very bad idea!
Clustered B+ Tree Used for Sorting

- Cost: root to the left-most leaf, then retrieve all leaf pages (Alternative 1)
- If Alternative 2 is used? Additional cost of retrieving data records: each page fetched just once.
- Fill factor of < 100% introduces a small overhead extra pages fetched

*Always better than external sorting!*
Unclustered B+ Tree Used for Sorting

- Alternative (2) for data entries: each data entry contains \textit{rid} of a data record. In general, one I/O per data record!

![Diagram of B+ Tree]

- Steering Blocks (Directs search)
- Leaf Blocks (Sequence set)
- Data Records
## External Sorting vs. Unclustered Index

<table>
<thead>
<tr>
<th>N</th>
<th>Sorting</th>
<th>p=1</th>
<th>p=10</th>
<th>p=100</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
<td>100</td>
<td>1,000</td>
<td>10,000</td>
</tr>
<tr>
<td>1,000</td>
<td>2,000</td>
<td>1,000</td>
<td>10,000</td>
<td>100,000</td>
</tr>
<tr>
<td>10,000</td>
<td>40,000</td>
<td>10,000</td>
<td>100,000</td>
<td>1,000,000</td>
</tr>
<tr>
<td>100,000</td>
<td>600,000</td>
<td>100,000</td>
<td>1,000,000</td>
<td>10,000,000</td>
</tr>
<tr>
<td>1,000,000</td>
<td>8,000,000</td>
<td>1,000,000</td>
<td>10,000,000</td>
<td>100,000,000</td>
</tr>
<tr>
<td>10,000,000</td>
<td>80,000,000</td>
<td>10,000,000</td>
<td>100,000,000</td>
<td>1,000,000,000</td>
</tr>
</tbody>
</table>

- $p$: # of records per page
- $B=1,000$ and block size=32 for sorting
- $p=100$ is the more realistic value.
Summary

- External sorting is important; DBMS may dedicate part of buffer pool just for sorting!
- External merge sort minimizes disk I/O cost:
  - Pass 0: Produces sorted runs of size $B$ (# buffer pages). Later passes: merge runs.
  - # of runs merged at a time depends on $B$, and block size.
  - Larger block size means less I/O cost per page.
  - Larger block size means smaller # runs merged.
  - In practice, # of runs rarely more than 2 or 3.
Summary, cont.

❖ Choice of internal sort algorithm may matter:
  ▪ Quicksort: Quick!
  ▪ Alternative sorts
    • guaranteed $N \log N$ on worst case data
    • stable (ties retain their original order)

❖ The best sorts are wildly fast:
  ▪ Despite 40+ years of research, we’re still improving!

❖ Clustered B+ tree is good for sorting; unclustered tree is usually very bad.
Next Time

Schema Refinement

...have a good break!